Cambridge Assessment International Education

Cambridge International A Level

MATHEMATICS **9709/32**

Paper 3 Pure Mathematics 3 **October/November 2020**

MARK SCHEME

Maximum Mark: 75

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the October/November 2020 series for most Cambridge IGCSE™, Cambridge International A and AS Level and Cambridge Pre-U components, and some Cambridge O Level components.

This document consists of **18** printed pages.

Generic Marking Principles

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptors for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1:
Marks must be awarded in line with:
- the specific content of the mark scheme or the generic level descriptors for the question
- the specific skills defined in the mark scheme or in the generic level descriptors for the question
- the standard of response required by a candidate as exemplified by the standardisation scripts.

GENERIC MARKING PRINCIPLE 2:
Marks awarded are always **whole marks** (not half marks, or other fractions).

GENERIC MARKING PRINCIPLE 3:
Marks must be awarded **positively**:
- marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate
- marks are awarded when candidates clearly demonstrate what they know and can do
- marks are not deducted for errors
- marks are not deducted for omissions
- answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.

GENERIC MARKING PRINCIPLE 4:
Rules must be applied consistently, e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.

GENERIC MARKING PRINCIPLE 5:
Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).

GENERIC MARKING PRINCIPLE 6:
Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

Mathematics Specific Marking Principles	
1	Unless a particular method has been specified in the question, full marks may be awarded for any correct method. However, if a calculation is required then no marks will be awarded for a scale drawing.
2	Unless specified in the question, answers may be given as fractions, decimals or in standard form. Ignore superfluous zeros, provided that the degree of accuracy is not affected.
3	Allow alternative conventions for notation if used consistently throughout the paper, e.g. commas being used as decimal points.
4	Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored (isw).
5	Where a candidate has misread a number in the question and used that value consistently throughout, provided that number does not alter the difficulty or the method required, award all marks earned and deduct just 1 mark for the misread.
6	Recovery within working is allowed, e.g. a notation error in the working where the following line of working makes the candidate's intent clear.

Mark Scheme Notes

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

Types of mark

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

DM or **DB** When a part of a question has two or more "method" steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly, when there are several B marks allocated. The notation DM or DB is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

FT Implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only.

- A or B marks are given for correct work only (not for results obtained from incorrect working) unless follow through is allowed (see abbreviation FT above).
- For a numerical answer, allow the A or B mark if the answer is correct to 3 significant figures or would be correct to 3 significant figures if rounded (1 decimal place for angles in degrees).
- The total number of marks available for each question is shown at the bottom of the Marks column.
- Wrong or missing units in an answer should not result in loss of marks unless the guidance indicates otherwise.
- Square brackets [ ] around text or numbers show extra information not needed for the mark to be awarded.

Abbreviations

AEF/OE	Any Equivalent Form (of answer is equally acceptable) / Or Equivalent
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
CAO	Correct Answer Only (emphasising that no 'follow through' from a previous error is allowed)
CWO	Correct Working Only
ISW	Ignore Subsequent Working
SOI	Seen Or Implied
SC	Special Case (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)
WWW	Without Wrong Working
AWRT	Answer Which Rounds To

Question	Answer	Marks	Guidance
1	State that $1+e^{-3x}=e^2$	**B1**	With no errors seen to that point
	Use correct method to solve an equation of the form $e^{-3x}=a$, where $a>0$, for x or equivalent	**M1**	$\left(e^{-3x}=6.389...\right)$ Evidence of method must be seen.
	Obtain answer $x=-0.618$ only	**A1**	Must be 3 decimal places
	Alternative method for question 1		
	State that $1+e^{-3x}=e^2$	**B1**	
	Rearrange to obtain an expression for e^x and solve an equation of the form $e^x=a$, where $a>0$, or equivalent	**M1**	$e^x=\sqrt[3]{\dfrac{1}{e^2-1}}$
	Obtain answer $x=-0.618$ only	**A1**	Must be 3 decimal places
		3	

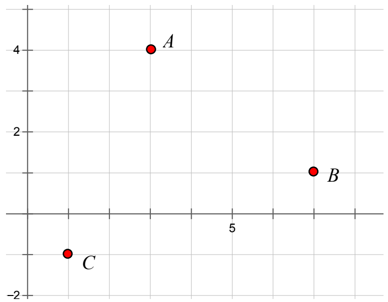
Question	Answer	Marks	Guidance
2(a)	State a correct unsimplified version of the x or x^2 or x^3 term	**M1**	For the given expression
	State correct first two terms $1+2x$	**A1**	
	Obtain the next two terms $-4x^2+\dfrac{40}{3}x^3$	**A1 + A1**	One mark for each correct term. ISW Accept $13\frac{1}{3}$ The question asks for simplified coefficients, so candidates should cancel fractions.
		4	
2(b)	State answer $\lvert x\rvert<\dfrac{1}{6}$	**B1**	OE. Strict inequality
		1	

Question	Answer	Marks	Guidance
3(a)	State or imply $y\log 2 = \log 3 - 2x\log 3$	**B1**	Accept $y\ln 2 = (1-2x)\ln 3$
	State that the graph of y against x has an equation which is linear in x and y, or is of the form $ay = bx + c$	**B1**	Correct equation. Need a clear statement/comparison with matching linear form.
	Clear indication that the gradient is $-\dfrac{2\ln 3}{\ln 2}$	**B1**	Must be exact. Any equivalent e.g. $-\dfrac{2\log_k 3}{\log_k 2}$, $\log_2 \dfrac{1}{9}$
		3	
3(b)	Substitute $y = 3x$ in an equation involving logarithms and solve for x	**M1**	
	Obtain answer $x = \dfrac{\ln 3}{\ln 72}$	**A1**	Allow M1A1 for the correct answer following decimals
		2	

Question	Answer	Marks	Guidance
4(a)	Use correct $\tan(A+B)$ formula and obtain an equation in $\tan\theta$	M1	e.g. $\dfrac{\tan\theta+\tan 60°}{1-\tan\theta\tan 60°}=\dfrac{2}{\tan\theta}$
	Use $\tan 60°=\sqrt{3}$ and obtain a correct horizontal equation in any form	A1	e.g. $\tan\theta\left(\tan\theta+\sqrt{3}\right)=2\left(1-\sqrt{3}\tan\theta\right)$
	Reduce to $\tan^2\theta+3\sqrt{3}\tan\theta-2=0$ correctly	A1	AG
		3	
4(b)	Solve the given quadratic to obtain a value for θ	M1	$\left(\tan\theta=\dfrac{-3\sqrt{3}\pm\sqrt{35}}{2}=0.3599,-5.556\right)$
	Obtain one correct answer e.g. $\theta=19.8°$	A1	Accept 1d.p. or better. If over-specified must be correct. 19.797...., 100.2029...
	Obtain second correct answer $\theta=100.2°$ and no others in the given interval	A1	Ignore answers outside the given interval.
		3	

Question	Answer	Marks	Guidance
5(a)	State $\dfrac{dx}{d\theta} = \sec^2\theta$ or $\dfrac{dy}{d\theta} = -2\sin\theta\cos\theta$	B1	CWO, AEF.
	Use $\dfrac{dy}{dx} = \dfrac{dy}{d\theta} \div \dfrac{dx}{d\theta}$	M1	
	Obtain $\dfrac{dy}{dx} = -2\sin\theta\cos^3\theta$ from correct working	A1	AG
	Alternative method for question 5(a)		
	Convert to Cartesian form and differentiate	M1	$y = \dfrac{1}{1+x^2}$
	$\dfrac{dy}{dx} = \dfrac{-2x}{\left(1+x^2\right)^2}$	A1	OE
	Obtain $\dfrac{dy}{dx} = -2\sin\theta\cos^3\theta$ from correct working	A1	AG
		3	

Question	Answer	Marks	Guidance
5(b)	Use correct product rule to obtain $\frac{\mathrm{d}}{\mathrm{d}\theta}\left(\pm 2\cos^3\theta\sin\theta\right)$	M1	Condone incorrect naming of the derivative For work done in correct context
	Obtain correct derivative in any form	A1	e.g. $\pm\left(-2\cos^4\theta+6\sin^2\theta\cos^2\theta\right)$
	Equate derivative to zero and obtain an equation in one trig ratio	A1	e.g. $3\tan^2\theta=1$, or $4\sin^2\theta=1$ or $4\cos^2\theta=3$
	Obtain answer $x=-\frac{1}{\sqrt{3}}$	A1	Or $-\frac{\sqrt{3}}{3}$
	Alternative method for question 5(b)		
	Use correct quotient rule to obtain $\frac{\mathrm{d}^2y}{\mathrm{d}x^2}$	M1	
	Obtain correct derivative in any form	A1	$\frac{-2\left(1+x^2\right)^2+2\times 2x\times 2x\left(1+x^2\right)}{\left(1+x^2\right)^4}$
	Equate derivative to zero and obtain an equation in x^2	A1	e.g. $6x^2=2$
	Obtain answer $x=-\frac{1}{\sqrt{3}}$	A1	
		4	

Question	Answer	Marks	Guidance
6(a)	Multiply numerator and denominator by 1 + i, or equivalent	**M1**	Must multiply out
	Obtain numerator 6 + 8i or denominator 2	**A1**	
	Obtain final answer $u = 3 + 4\mathrm{i}$	**A1**	
	Alternative method for question 6(a)		
	Multiply out $(1-i)(x+iy)=7+i$ and compare real and imaginary parts	**M1**	
	Obtain $x+y=7$ or $y-x=1$	**A1**	
	Obtain final answer $u = 3 + 4\mathrm{i}$	**A1**	
		3	
6(b)	Show the point A representing u in a relatively correct position	**B1 FT**	The FT is on $xy \neq 0$.
	Show the other two points B and C in relatively correct positions: approximately equal distance above / below real axis	**B1**	 Take the position of A as a guide to ‘scale’ if axes not marked
		2	

Question	Answer	Marks	Guidance
6(c)	State or imply arg(1 – i) = $-\frac{1}{4}\pi$	**B1**	Arg$C$
	Substitute exact arguments in arg(7 + i) – arg(1 – i) = arg u	**M1**	Must see a statement about the relationship between the Args e.g. Arg$A$ = Arg$B$ – Arg$C$ or equivalent exact method
	Obtain $\tan^{-1}\left(\frac{4}{3}\right) = \tan^{-1}\left(\frac{1}{7}\right) + \frac{1}{4}\pi$ correctly	**A1**	Obtain given answer correctly from *their* $u = k(3+4i)$
		3	

Question	Answer	Marks	Guidance
7(a)	Correct separation of variables	B1	$\int \sec^2 2x\,\mathrm{d}x = \int \mathrm{e}^{-3t}\,\mathrm{d}t$ Needs correct structure
	Obtain term $-\frac{1}{3}\mathrm{e}^{-3t}$	B1	
	Obtain term of the form $k\tan 2x$	M1	From correct working
	Obtain term $\frac{1}{2}\tan 2x$	A1	
	Use $x = 0$, $t = 0$ to evaluate a constant, or as limits in a solution containing terms of the form $a\tan 2x$ and $b\mathrm{e}^{-3t}$, where $ab \neq 0$	M1	
	Obtain correct solution in any form	A1	e.g. $\frac{1}{2}\tan 2x = -\frac{1}{3}\mathrm{e}^{-3t} + \frac{1}{3}$
	Obtain final answer $x = \frac{1}{2}\tan^{-1}\left(\frac{2}{3}\left(1-\mathrm{e}^{-3t}\right)\right)$	A1	
		7	
7(b)	State that x approaches $\frac{1}{2}\tan^{-1}\left(\frac{2}{3}\right)$	B1 FT	Correct value. Accept $x \to 0.294$ The FT is dependent on letting $\mathrm{e}^{-3t} \to 0$ in a solution containing e^{-3t}.
		1	

Question	Answer	Marks	Guidance
8(a)	Obtain $\overrightarrow{AB} = \begin{pmatrix} 2 \\ -2 \\ -4 \end{pmatrix}$ and $\overrightarrow{CD} = \begin{pmatrix} 2 \\ 1 \\ 1 \end{pmatrix}$	**B1**	Or equivalent seen or implied
	Use the correct process for calculating the modulus of both vectors to obtain AB and CD	**M1**	$AB = \sqrt{24}$, $CD = \sqrt{6}$
	Using exact values, verify that $AB = 2CD$	**A1**	Obtain given statement from correct work Allow from $BA = 2DC$, OE
		3	
8(b)	Use the correct process to calculate the scalar product of the relevant vectors (*their* $\overrightarrow{AB}$ and $\overrightarrow{CD}$)	**M1**	$\begin{pmatrix} 2 \\ -2 \\ -4 \end{pmatrix}$ and $\begin{pmatrix} 2 \\ 1 \\ 1 \end{pmatrix}$ or $\begin{pmatrix} 2 \\ -2 \\ -4 \end{pmatrix}$ and $\begin{pmatrix} 4 \\ 2 \\ 2 \end{pmatrix}$
	Divide the scalar product by the product of the moduli and evaluate the inverse cosine of the result	**M1**	
	Obtain answer $99.6°$ (or 1.74 radians) or better	**A1**	Do not ISW if go on to subtract from 180° (99.594…, 1.738...) Accept 260.4°
		3	

<table>
<tr><th>Question</th><th>Answer</th><th>Marks</th><th>Guidance</th></tr>
<tr><td>8(c)</td><td>State correct vector equations for AB and CD in any form,
e.g. $(\mathbf{r}=)\begin{pmatrix}2\\1\\5\end{pmatrix}+\lambda\begin{pmatrix}2\\-2\\-4\end{pmatrix}$ and $(\mathbf{r}=)\begin{pmatrix}1\\1\\2\end{pmatrix}+\mu\begin{pmatrix}2\\1\\1\end{pmatrix}$</td><td>B1ft</td><td>Follow their $\overrightarrow{AB}$ and $\overrightarrow{CD}$
Alternative: $(\mathbf{r}=)\begin{pmatrix}4\\-1\\1\end{pmatrix}+\lambda\begin{pmatrix}2\\-2\\-4\end{pmatrix}$ and $(\mathbf{r}=)\begin{pmatrix}3\\2\\3\end{pmatrix}+\mu\begin{pmatrix}2\\1\\1\end{pmatrix}$</td></tr>
<tr><td></td><td>Equate at least two pairs of components of their lines and solve for λ or for μ</td><td>M1</td><td></td></tr>
<tr><td></td><td>Obtain correct pair of values from correct equations</td><td>A1</td><td>Alternatives when taking A or B as point on line
<table>
<tr><th>A</th><th>λ</th><th>μ</th><th></th><th>B</th><th>λ</th><th>μ</th><th></th></tr>
<tr><td>ij</td><td>$-\frac{1}{6}$</td><td>$\frac{1}{3}$</td><td>$\frac{17}{3}\neq\frac{7}{3}$</td><td>ij</td><td>$-\frac{7}{6}$</td><td>$-\frac{2}{3}$</td><td>$\frac{17}{3}\neq\frac{7}{3}$</td></tr>
<tr><td>ik</td><td>$\frac{1}{2}$</td><td>1</td><td>$0\neq 2$</td><td>ik</td><td>$-\frac{1}{2}$</td><td>0</td><td>$0\neq 2$</td></tr>
<tr><td>jk</td><td>$\frac{3}{2}$</td><td>-3</td><td>$5\neq -5$</td><td>jk</td><td>$\frac{1}{2}$</td><td>-4</td><td>$5\neq -5$</td></tr>
</table></td></tr>
<tr><td></td><td>Verify that all three equations are not satisfied and that the lines do not intersect</td><td>A1</td><td>CWO with conclusion
e.g. $\frac{17}{3}\neq\frac{7}{3}$ or $\frac{17}{3}=\frac{7}{3}$ is inconsistent or equivalent</td></tr>
<tr><td></td><td></td><td>4</td><td></td></tr>
</table>

Question	Answer	Marks	Guidance
9(a)	State or imply the form $\dfrac{A}{3x+2}+\dfrac{Bx+C}{x^2+4}$	**B1**	
	Use a correct method for finding a constant	**M1**	
	Obtain one of $A = 3$, $B = -1$, $C = 3$	**A1**	
	Obtain a second value	**A1**	
	Obtain the third value	**A1**	
		5	
9(b)	Integrate and obtain $\ln(3x+2)\ldots$	**B1 FT**	The FT is on A
	State a term of the form $k\ln\left(x^2+4\right)$.	**M1**	From $\int \dfrac{\lambda x}{x^2+4}\mathrm{d}x$
	$\ldots-\dfrac{1}{2}\ln\left(x^2+4\right)\ldots$	**A1 FT**	The FT is on B
	$\ldots+\dfrac{3}{2}\tan^{-1}\dfrac{x}{2}$	**B1 FT**	The FT is on C
	Substitute limits correctly in an integral with at least two terms of the form $a\ln(3x+2)$, $b\ln\left(x^2+4\right)$ and $c\tan^{-1}\left(\dfrac{x}{2}\right)$, and subtract in correct order	**M1**	Using terms that have been obtained correctly from completed integrals
	Obtain answer $\dfrac{3}{2}\ln 2+\dfrac{3}{8}\pi$, or exact 2-term equivalent	**A1**	
		6	

Question	Answer	Marks	Guidance
10(a)	Use correct product rule	**M1**	
	Obtain correct derivative in any form	**A1**	e.g. $\frac{dy}{dx}=\frac{1}{2\sqrt{x}}\cos x-\sqrt{x}\sin x$. Accept in a or in x
	Equate derivative to zero and obtain $\tan a=\frac{1}{2a}$	**A1**	Obtain given answer from correct working**.** The question says 'show that ..' so there should be an intermediate step e.g. $\cos x=2x\sin x$. Allow $\tan x=\frac{1}{2x}$
		3	
10(b)	Use the iterative process correctly at least once (get one value and go on to use it in a second use of the formula)	**M1**	Must be working in radians Degrees gives 1, 12.6039, 5.4133, ... M0
	Obtain final answer 3.29	**A1**	Clear conclusion
	Show sufficient iterations to at least 4 d.p.to justify 3.29, or show there is a sign change in the interval (3.285, 3.295)	**A1**	$3, 3.3067, 3.2917, 3.2923$ Allow more than 4d.p. Condone truncation.
		3	

Question	Answer	Marks	Guidance
10(c)	State or imply the indefinite integral for the volume is $\pi\int\left(\sqrt{x}\cos x\right)^2 \mathrm{d}x$	B1	[If π omitted, or 2π or $\frac{1}{2}\pi$ used, give B0 and follow through. 4/6 available]
	Use correct cos $2A$ formula, commence integration by parts and reach $x(ax+b\sin 2x)\pm\int ax+b\sin 2x\mathrm{d}x$	*M1	Alternative: $\frac{x^2}{4}+\frac{x}{4}\sin 2x-\int\frac{1}{4}\sin 2x\mathrm{d}x$
	Obtain $x(\frac{1}{2}x+\frac{1}{4}\sin 2x)-\int\frac{1}{2}x+\frac{1}{4}\sin 2x\,\mathrm{d}x$, or equivalent	A1	
	Complete integration and obtain $\frac{1}{4}x^2+\frac{1}{4}x\sin 2x+\frac{1}{8}\cos 2x$	A1	OE
	Substitute limits $x=0$ and $x=\frac{1}{2}\pi$, having integrated twice	DM1	$\frac{\pi}{2}\left[\frac{\pi^2}{8}+0-\frac{1}{4}-0-0-\frac{1}{4}\right]$
	Obtain answer $\frac{1}{16}\pi\left(\pi^2-4\right)$, or exact equivalent	A1	CAO
		6	